Assignment

Solid State Physics IV

October 27, 2025

Instructions

- Show all steps clearly and justify your answers.
- Submit as a single PDF file before the due date, December 15, 2025, via email to hirschberger@ap.t.u-tokyo.ac.jp

Problem 1: Kubo formula

In the lecture, we proposed a perturbed wavefunction

$$|n\rangle_E = |n\rangle + \sum_{m(\neq n)} \frac{\langle m| - qEy\, |n\rangle}{E_n - E_m}\, |m\rangle \tag{1}$$

under the application of an electric field E. Show the various steps leading to the equation for the Hall conductivity,

$$\begin{aligned}\sigma_{xy} &= \frac{1}{E}\left[\langle j_x\rangle_E - \langle j_x\rangle_{E=0}\right] \\ &= -\frac{i\hbar q^2}{L^2}\sum_{n\neq m}\left[\frac{\langle n|\, v_x\, |m\rangle \langle m|\, v_y\, |n\rangle}{(E_n - E_m)^2} - \frac{\langle n|\, v_y\, |m\rangle \langle m|\, v_x\, |n\rangle}{(E_n - E_m)^2}\right].\end{aligned} \tag{2}$$

Problem 2: Kubo formula for Bloch functions

Use the Bloch functions $|u_{n\mathbf{k}}\rangle$, obeying

$$|\psi_n(\mathbf{k})\rangle = e^{i\mathbf{k}\cdot\mathbf{x}}\, |u_{n\mathbf{k}}\rangle \tag{3}$$

$$\hat{\mathcal{H}}_{\mathbf{k}}\, |u_{n\mathbf{k}}\rangle = E_{n\mathbf{k}}\, |u_{n\mathbf{k}}\rangle \tag{4}$$

with the Hamiltonian $\hat{\mathcal{H}}_{\mathbf{k}}$ as defined in the lecture. Further use

$$\langle u_{n\mathbf{k}}|\, \hat{\mathbf{v}}\, |u_{m\mathbf{k}}\rangle = \frac{1}{\hbar}\frac{\partial E_{n\mathbf{k}}}{\partial \mathbf{k}}\delta_{nm} + \frac{1}{\hbar}\left(E_{m\mathbf{k}} - E_{n\mathbf{k}}\right)\left\langle u_{n\mathbf{k}} \middle| \frac{\partial u_{n\mathbf{k}}}{\partial \mathbf{k}}\right\rangle \tag{5}$$

$$0 = \nabla_{\mathbf{k}}\, \langle u_{n\mathbf{k}}|\, u_{m\mathbf{k}}\rangle = \langle \nabla_{\mathbf{k}} u_{n\mathbf{k}}|\, u_{m\mathbf{k}}\rangle + \langle u_{n\mathbf{k}}|\, \nabla_{\mathbf{k}} u_{m\mathbf{k}}\rangle \tag{6}$$

where the latter relation is valid for $n \neq m$ *and* $n = m$. Derive the TKNN equation.