Problem Assignment

Condensed Matter Physics IV

The University of Tokyo

Max Hirschberger

hirschberger@ap.t.u-tokyo.ac.jp

http://www.qpec.t.u-tokyo.ac.jp/hirschberger/

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Instructions: Please solve the problem and return it to me (hirschberger@ap.t.u-tokyo.ac.jp) by January 29th, 17:30 Japanese Standard Time at the latest. Written comments should be in English, but you can type the solution or hand-write it as you choose.

One-dimensional topological insulator

We re-examine topological insulators through a one-dimensional lattice model (lattice spacing: $a = 1$ in dimensionless units). Let the Hamiltonian represented in one-dimensional k_x-space be

$$\mathcal{H}(k_x) = \tau_y \sin k_x + \tau_x \left[m + r(1 - \cos k_x)\right] \tag{1}$$

where the Pauli matrices are

$$\tau_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \tau_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{2}$$

and we introduced two parameters $r > 0$ and m.

Assignment:

1. Find the two eigenstates $|\pm, k_x\rangle$ and corresponding eigenenergies $E_\pm$. In the case of $r = 1$, plot the dispersion in the first Brillouin zone for $m = -3, -2, -1, 0$, and 1. Can you identify the band inversion at the TRIM $k_x = 0, \pi$? Carefully consider the definition of the inner product by which the eigenvectors are orthogonal.

2. Let us represent the eigenstates $|\pm, k_x\rangle$ as $2^{-1/2}\left(1, \pm e^{i\chi(k_x)}\right)$. Then, show that $\tan\chi(k_x) = \sin k_x / \left[m + r(1 - \cos k_x)\right]$. Plot the phase $\chi(k_x)$ within the first Brilloin zone for the three cases $m < -2r$, $-2r < m < 0$, and $m > 0$.

3. Consider the topological winding number $\nu = 0, 1$ according to the expression

$$\nu = -\frac{1}{\pi}\int_{-\pi}^{\pi} A(k_x) dk_x \tag{3}$$

where the Berry connection is

$$A(k_x) = -i\, \langle \pm, k_x | \frac{\partial}{\partial k_x} | \pm, k_x \rangle \tag{4}$$

Obtain ν for the abovementioned three regimes of m.

4. At the TRIM $k_x = \Lambda_i = 0, \pi$, the Hamiltonian reduces to $\mathcal{H}(\Lambda_i) = \tau_x \left[m + r(1 - \cos\Lambda_i)\right]$. The quantity $\delta(\Lambda_i) = \mathrm{Pf}\left[w(\Lambda_i)\right] / \sqrt{\det\left[w(\Lambda_i)\right]}$ then only depends on the sign of $m + r(1 - \cos\Lambda_i)$. Discuss a condition for classifying the system into topological or non-topological, depending on the parameter m.